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# DISTRIBUTION OF ELECTRON CONCENTRATION IN A DISCHARGE WITH NONUNIFORM IONIZATION OVER THE CROSS SECTION

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We have obtained an analytic solution of the charged particle balance equation for the plasma of the positive column of a discharge, with allowance for the radial variation of the ionization rate.

To find the radial distribution of the electron concentration in the positive column it is necessary to solve the particle balance differential equation with variable coefficients [1]. Solutions of this equation alone [2, 3], and also of a system of equations describing the properties of the discharge [4-7], were obtained by numerical and approximate methods. In the present article we obtain the distribution of the electron concentration by an analytic method.

We assume that the plasma of a glow discharge consists of neutral particles, singly charged positive ions, and electrons. Charged particles are formed by direct ionization, and disappear by radial ambipolar diffusion with subsequent wall recombination. The plasma is quasineutral, and the discharge parameters are uniform in the axial direction and axisymmetric. The charged particle balance for an element of volume $2\pi rR^2dr \times 1$ is described by the familiar differential equation

$$\frac{1}{r}\frac{d}{dr}\left(rD_a\frac{dn}{dr}\right)+\nu R^2n=0. \tag{1}$$

The boundary conditions for this equation are generally written in the form

$$n(1)=0,\ \left(\frac{dn}{dr}\right)_{r=0}=0. \tag{2}$$

For a constant temperature of the gas over the cross section of the discharge chamber, as assumed in the Schottky theory [8], the coefficients $D_a$ and $\nu$ do not depend on the spatial coordinate. Actually, there is a certain nonuniformity of the gas temperature in the radial direction which depends on the strength of the discharge, the pressure of the gas, the conditions on the boundary surface, etc. Taking account of the temperature nonuniformity of the gas leads to coefficients $D_a(r)$ and $\nu(r)$ which depend on the radial coordinate. In view of the strong dependence of the ionization rate on the ratio E/N, its relative change along the radius can exceed the corresponding change of the coefficient of ambipolar diffusion by an order of magnitude for the same nonuniformity of the gas temperature. Therefore, in the first approximation we assume that the coefficient of ambipolar diffusion is constant and equal to a certain average for the temperature range considered. We assume that the ionization rate varies parabolically with the radius $\nu = \nu_R[1 + (1 - r^2)\alpha^2]$, where $\alpha$ takes account of the degree of nonuniformity of the ionization rate. Then Eq. (1) is written in the form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dn}{dr}\right)+\mu^2[1+(1-r^2)\alpha^2]n=0, \tag{3}$$

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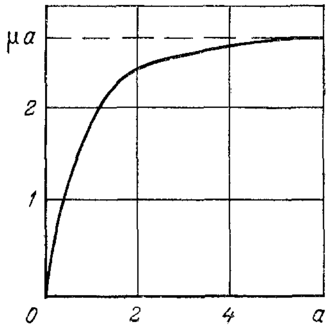


Fig. 1

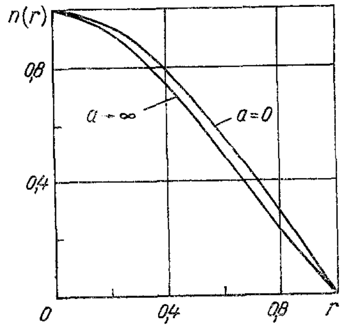


Fig. 2

Fig. 1. Graph of $\mu a$ = f($a$).

Fig. 2. Radial distributions of the electron concentration calculated for two values of the parameter $a$.

where $\mu^2 = \nu_R R^2/D_a$. Thus, the problem is reduced to a Sturm–Liouville problem of finding the eigenvalues $\mu$ as a function of the parameter $a$ and the corresponding eigenfunctions which satisfy Eq. (3) and condition (2). The transformation

$$n(r) = K(y)\exp(-y/2),\quad y = \mu a r^2 \tag{4}$$

reduces Eq. (3) to the form

$$y\frac{d^2K}{dy^2} + (1-y)\frac{dK}{dy} + \left(\frac{\mu a}{4a^2} + \frac{\mu a}{4} - \frac{1}{2}\right)K = 0. \tag{5}$$

The solution of this differential equation which is finite at r = 0 is the confluent hypergeometric function

$$K(y) = M\left(\frac{1}{2} - \frac{\mu a}{4a^2} - \frac{\mu a}{4},\ 1,\ \mu a r^2\right). \tag{6}$$

The eigenvalues $\mu = f(a)$ are determined from the solution of the equation

$$M\left(\frac{1}{2} - \frac{\mu a}{4a^2} - \frac{\mu a}{4},\ 1,\ \mu a\right) = 0. \tag{7}$$

These values can be calculated by using data on the zeros of the function M(α, 1, x) in [9]. Calculations show that with an increase in the parameter $a$ the values of $\mu$ for which condition (7) is satisfied decrease and approach zero. However, the product $\mu a$ increases. The calculated dependence of this product on the parameter $a$ is shown in Fig. 1. As $a \to \infty$ the value of $\mu a$ approaches a limit which is determined in the following way. In the limiting case $a \to \infty$, $\mu \to 0$, Eq. (3) takes the form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dn}{dr}\right) + (\mu a)^2_{\lim}(1-r^2)\,n = 0. \tag{8}$$

This differential equation is known in the theory of convective heat transfer as the Graetz–Nusselt problem. The eigenvalue $(\mu a)_{\lim} = 2.704$ and the eigenfunction satisfying Eq. (8) and condition (2) were calculated and reported in [10]. Taking account of condition (2), we write the general solution of Eq. (3) in the form

$$n(r) = \exp\left(-\frac{\mu a r^2}{2}\right) M\left(\frac{1}{2} - \frac{\mu a}{4a^2} - \frac{\mu a}{4},\ 1,\ \mu a r^2\right). \tag{9}$$

Figure 2 shows profiles of the electron concentration calculated for the two limiting cases $a = 0$ and $a \to \infty$. For other values of the parameter $a$, $0 < a < \infty$, the radial distributions of charged particles will lie between the curves of Fig. 2.

The solution (9) can also be written in the form of the

$$n(r) = \sum_{n=0}^{\infty} b_{2n} r^{2n}, \tag{10}$$